Rational Equations and Functions

Algebra II

Chapter 8

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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8.1 Model Inverse and Joint Variation

- Direct Variation: y = ax
  - x ↑, y ↑

- Inverse Variation: $y = \frac{a}{x}$
  - x ↑, y ↓

- Joint Variation: y = axz
  - y depends on both x and z

a is the constant of variation

8.1 Model Inverse and Joint Variation

- What type of variation is each of the following?
  - ∗ xy = 48

  - ∗ 2y = x

  - ∗ y = 2x + 3

8.1 Model Inverse and Joint Variation

- Solving Variations
  - ∗ Plug in x and y to find a
  - ∗ Plug in a and the other value and solve

- y varies inversely as x. When x = 2, y = 6. Write an equation relating x and y. Then find y when x = 4.

8.1 Model Inverse and Joint Variation

- Checking data for variation
  - ∗ Plug each of the data points in one of the variation equations to find a
  - ∗ If the a stays the same, the data has that type of variation
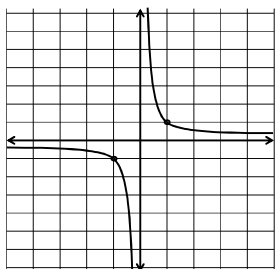- What type of variation?

x	2	4	8
y	8	4	2

8.1 Model Inverse and Joint Variation

- Writing variations from sentences
  - * y varies directly with x and inversely with z^2

  - * z varies jointly with x^2 and y

  - * y varies inversely with x and z

8.2 Graph Simple Rational Functions

- Rational Functions
  - * Functions written as a fraction with x in the denominator
  - * $y = \frac{1}{x}$
- Shape called hyperbola
- Asymptotes
  - * Horizontal: x-axis
  - * Vertical: y-axis

8.2 Graph Simple Rational Functions

- General form
  - * $y = \frac{a}{x-h} + k$
    - * a → stretches vertically (multiplies y-values)
    - * h → moves right
    - * k → moves up
- How is $y = \frac{2}{x+3} + 4$ transformed from $y = \frac{1}{x}$?

8.2 Graph Simple Rational Functions

- How to find asymptotes
  - * Vertical
    - * Make the denominator = 0 and solve for x

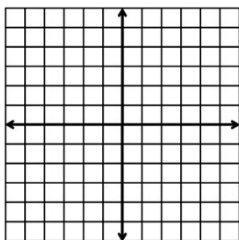## 8.2 Graph Simple Rational Functions

- * Horizontal
  - * Substitute a very large number for x and estimate y
- * Or
  - * Find the degree of numerator (N)
  - * Find the degree of denominator (D)
  - * If N < D, then y = 0
  - * If N = D, then y = leading coefficients
  - * If N > D, then no horizontal asymptote
- Find the asymptotes for $y = \frac{2x}{3x-6}$

8.2 Graph Simple Rational Functions

- Domain
  - * All x's except for the vertical asymptotes
- Range
  - * All the y's covered in the graph
  - * Usually all y's except for horizontal asymptote

8.2 Graph Simple Rational Functions

- Graph by finding asymptotes and making a table

- Graph $y = \frac{2}{x+3} + 4$

8.3 Graph General Rational Functions

- Find the asymptotes
  - Simplify first
    - Factor and cancel entire factors
  - Vertical
    - take the denominator = 0 and solve for x

8.3 Graph General Rational Functions

- Horizontal
  - Substitute a very large number for x and estimate y
- Or
  - Find the degree of numerator (N)
  - Find the degree of denominator (D)
  - If N < D, then y = 0
  - If N = D, then y = leading coefficients
  - If N > D, then no horizontal asymptote
- Find the asymptotes for $y = \frac{2x^2 + x}{x^2 - 1}$
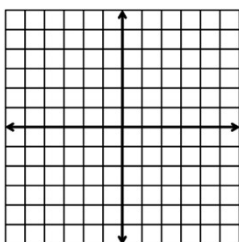
8.3 Graph General Rational Functions

- How to find x-intercepts
  - Let $y = 0$
    - If $y = \frac{numerator}{denominator} = 0$
    - Only happens if numerator = 0
- How to find y-intercepts
  - Let $x = 0$ and simplify

8.3 Graph General Rational Functions

- To graph rational functions
  - Find the asymptotes
  - Make a table of values around the vertical asymptotes
  - Graph the asymptotes and points
  - Start near an asymptote, go through the points and end near another asymptote
    - Each graph will have several sections
    - NEVER cross a vertical asymptote

8.3 Graph General Rational Functions

- Graph $y = \frac{2x^2+x}{x^2-1}$

8.4 Multiply and Divide Rational Expressions

- Simplified form → numerator and denominator can have no common factors

- Steps to simplify
  - Factor numerator and denominator
  - Cancel any common factors

8.4 Multiply and Divide Rational Expressions

Simplify

- $\dfrac{x^2-5x-6}{x^2-1}$

- $\dfrac{x^3+5x^2+6x}{x^3+2x^2}$

8.4 Multiply and Divide Rational Expressions

- Multiplying Rational Expressions
  - Factor numerators and denominators
  - Multiply across top and bottom
  - Cancel factors

8.4 Multiply and Divide Rational Expressions

- $\frac{3x-27x^3}{3x^2-2x-1} \cdot \frac{3x^2-4x+1}{3x}$
- $\frac{x+2}{27x^3+8} \cdot (9x^2 - 6x + 4)$

8.4 Multiply and Divide Rational Expressions

- Dividing Rational Expressions
  - ∗ Take reciprocal of divisor
  - ∗ Multiply
- $\frac{3}{4x-8} \div \frac{x^2+3x}{x^2+x-6}$

8.4 Multiply and Divide Rational Expressions

- Combined Operations
  - ∗ Do the first two operations
  - ∗ Use that result with the next operation

8.5 Add and Subtract Rational Expressions

- Adding and Subtracting
  - ∗ Need least common denominator (LCD)
    - ∗ If LCD already present, add or subtract numerators only

  - ∗ To get fractions with LCD
    - ∗ Factor all denominators
    - ∗ LCD is the common factors times the unique factors
    - ∗ Whatever you multiply the denominator by, multiply the numerator also

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8.5 Add and Subtract Rational Expressions

- $\frac{3}{2x} - \frac{7}{2x}$

- $\frac{3x}{x-4} + \frac{6}{x-4}$

8.5 Add and Subtract Rational Expressions

- $\frac{4}{3x^2} + \frac{x}{6x^3 + 3x^2}$

- $\frac{x+1}{x^2 + 6x + 9} - \frac{1}{x^2 - 9}$

8.5 Add and Subtract Rational Expressions

- Simplifying Complex Fractions
  * Fractions within fractions
  * Follow order of operations (groups first)
  * Divide

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8.5 Add and Subtract Rational Expressions

- $$\dfrac{\dfrac{3}{x-4}}{\dfrac{1}{x-4}+\dfrac{3}{x+1}}$$

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8.6 Solve Rational Equations

- Only when the = sign is present!!!

- Method 1: simplify both sides and cross multiply

- Method 2:
- Multiply both sides by LCD to remove fractions
- Solve
- Check answers

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8.6 Solve Rational Equations

- $\frac{3}{x} - \frac{1}{2} = \frac{12}{x}$

- $\frac{5x}{x+1} = 4 - \frac{5}{x+1}$

8.6 Solve Rational Equations

- $\frac{3x-2}{x-2} = \frac{6}{x^2-4} + 1$

- $\frac{3}{x^2+4x} = \frac{1}{x+4}$